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### ABSTRACT:

The gray system theory which proposed by the Chinese researcher Julong Deng is the theory that deals with the uncertain information of phenomena, according to which few data can be used to create models without taking into account the distribution of those data. Therefore, this theory has been widely used in the field of image processing, time-series prediction, system optimization, control and decision, etc. Gray system prediction is one of the most important components of gray system theory that works to find rules for system development through certain data processing procedures in order to predict the future of the system scientifically and quantitatively.

When the time series data have large fluctuations and a greater oscillation range, it is difficult to build an appropriate model to describe the behavior of that time series, in this case we can think of predicting the wave of behavioral data development in the future based on the original wave of data, this kind of prediction is called wave prediction, and its modeling process is obvious and simple, so wave prediction is gaining high application value in many fields.

In this paper, gray wave prediction models will be used to predict crude oil production in Iraq based on the time series data of the monthly production rate, whose observations suffer from large fluctuations.

**Keywords:** Time series, Grey Model, Grey Wave model, Contour lines.

### استعمال النماذج الموجية الرمادية للتنبؤ بإنتاج النفط الخام في العراق المستخلص

نظرية النظام الرمادي المقترحة من قبل الباحث الصيني Julong Deng هي النظرية التي تتعامل مع المعلومات غير المؤكدة للظواهر إذ يمكن بموجبها استعمال بعض البيانات لإنشاء نماذج دون مراعاة توزيع تلك البيانات. لذا تم استعمال هذه النظرية على نطاق واسع في مجال معالجة الصور والتنبؤ في مجال السلاسل الزمنية وتحسين النظام والتحكم والقرار وما إلى ذلك. يعد التنبؤ بالنظام الرمادي أحد أهم المكونات في نظرية النظام الرمادي الذي يعمل على إيجاد قواعد تطوير النظام من خلال إجراءات معينة لمعالجة البيانات من أجل التنبؤ بمستقبل النظام علمياً وكمياً.

عندما تكون بيانات السلسلة الزمنية ذات تقلبات كبيرة ونطاق تأرجحها أكبر، يكون من الصعب بناء نموذج مناسب لوصف سلوك تلك السلسلة الزمنية، في هذه الحالة يمكننا التفكير في التنبؤ بموجة تطوير البيانات السلوكية في المستقبل بناءً على موجة البيانات الأصلية، ويسمى هذا النوع من التنبؤ بالتنبؤ بالموجات. يعد التنبؤ بالموجات أحد المحتويات المهمة لنظام التنبؤ الرمادي، وعملية النمذجة الخاصة به واضحة وبسيطة، لذا فإن التنبؤ بالموجات يكتسب قيمة عالية للتطبيق في العديد من المجالات.

في هذا البحث سيتم استعمال نماذج التنبؤ الموجية الرمادية للتنبؤ بإنتاج النفط الخام في العراق بالاعتماد على بيانات السلسلة الزمنية لمعدل الانتاج الشهري والتي تعاني مشاهداتها من التقلبات الكبيرة.

## 1. Introduction

In systems theory, colors are used to express the availability of information or not for the studied system, as the white color is used to express the availability of complete information for the system, so it is called the white system, while the black color used to express the complete lack of information and is called the black system, while the system is called the gray system if the information about it is tainted by uncertainty or accuracy, and for this system, we find that some of the information is known and the other part is unknown.

Gray system theory can be defined as a research in uncertainties due to its use of little data and information. This theory is used with little and uncertain data for small samples of data and partial and weak information as the main research inputs.

The gray system was proposed by the Chinese scientist Deng Ju-Long in 1982 [8]. Which is adopted as a method for forecasting non-linear time series, and this system has been developed by researchers and adopted in various fields, economic, industrial and natural phenomena.

Gray models are useful for predicting the behavior of a monotonous time series, meaning that it is characterized as ascending or descending pattern, as it gives predictions with high accuracy, but it is inefficient in the case of time series with periodic and transverse fluctuations, which prompted researchers to think about predicting the wave of data behavior in the future based on the wave of the original data and this type of prediction is called the wave prediction, which is one of the most important gray forecasting and the adopted model is called the gray wave model and it is of interest in this research.

This research has been divided into five main sections, the first includes the introduction, the second includes the gray prediction model and clarification of its mechanism of action and steps for its construction, the third includes the gray wave model and clarification of the justifications for its existence and steps for its construction, and the fourth section includes the applied aspect of the research where the application is applied to two series of data, and finally the section fifth for conclusions.

## 2. Gray Forecasting Model

The gray forecasting model is one of the quantitative models used in prediction, which depends on the gray generation function, where the cumulative generation process is used to form differential equations that have properties that require less data, so this model appears as an effective tool for obtaining valuable information from small data sets [9]. This model is referred to as  $GM(p,q)$ , as the symbol  $p$  indicates the order of the differential equation and the symbol  $q$  indicates the number of variables included in the model [4].

As a special case of the gray forecasting model  $GM(p,q)$ , the  $GM(1,1)$  model, which is characterized by its simplicity and accuracy, as well as being the most widely used despite the presence of several other types, but most researchers focus on using this model for its computational efficiency, and this model is

described as A first-order linear gray model that consists of a first-order differential equation with only one variable [18].

The idea of GM(1,1) can be summarized in transforming the original data series of random nature into a regular data series to achieve an arrangement that meets the requirements of the forecasting model, where the raw data that suffers from randomness undergoes a cumulative assembly process called accumulative generation operator (AGO), then the differential equation of the model is solved depending on the data series resulting from the accumulation process to get the expected value, then the inverse of the accumulation process is applied to get the predictive value [14]. Thus, we can say that prediction in the gray models includes three main processes: the cumulative generation process, the gray modelling, and the inverse accumulated generating operation [5], and these steps will be explained as follows [1], [10], [16]:

I. Preparation the original time series data assuming that it is represented as follows [1]:

$$x^{(0)} = (x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n)) \quad , n \geq 4 \quad \dots (1)$$

Where n represent the time series length.

And then using the accumulative generation operator to create a new sequence for the original data series, as follows:

AGO:  $x^{(0)} \rightarrow x^{(1)}$

$$x^{(1)} = [x^{(1)}(1), x^{(1)}(2), \dots, x^{(1)}(n)] \quad \dots (2)$$

Where  $x^{(1)}$  represents the new series after cumulative additions, which is obtained as follows:

$$x^{(1)}(m) = \sum_{k=1}^m x^{(0)}(k) \quad , m = 1, 2, \dots, n \quad \dots (3)$$

The effect of randomness and irregularity in the original data series will be weak and decreases after the accumulative generation operator takes place [6].

II. Generate a new time series denoted by  $z(1)$ , whose values are the averages of the adjacent values of the time series  $x^{(1)}$  which generated using the accumulative generation operator eq.(2), as following:

$$z^{(1)}(m) = 0.5 (x^{(1)}(m) + x^{(1)}(m-1)) \quad , m = 2, 3, \dots \quad \dots (4)$$

III. Create a grey differential equation as shown in the following equation which represents the first-order differential equation through which we can model  $x^{(1)}$ :

$$x^{(0)}(m) + \alpha z^{(1)}(m) = \beta \quad , m = 2, 3, \dots, n \quad \dots (5)$$

Where:  $\alpha$ : the developed coefficient.  $\beta$ : the grey controlled variable.

Equation (3) can be reformulated as follow:

$$x^{(0)}(m) = -\alpha z^{(1)}(m) + \beta \quad , m = 2, 3, \dots, n \quad \dots (6)$$

We can formulate equation (6) in the form of matrices, as follows:

$$\begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix} = \begin{bmatrix} -z^{(0)}(2) & 1 \\ -z^{(0)}(3) & 1 \\ \vdots & \vdots \\ -z^{(0)}(n) & 1 \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \quad \dots (7)$$

The parameters of the gray model are estimated in equation (7) using the least square method, as follows:

$$\hat{p} = \begin{bmatrix} \hat{\alpha} \\ \hat{\beta} \end{bmatrix} = (Z'Z)^{-1}Z'y$$

Where:

$$Z = \begin{bmatrix} -z^{(0)}(2) & 1 \\ -z^{(0)}(3) & 1 \\ \vdots & \vdots \\ -z^{(0)}(n) & 1 \end{bmatrix}, \quad y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}$$

After getting the values  $\hat{\alpha}$  and  $\hat{\beta}$  the response time values  $\hat{x}^{(1)}(m)$  are calculated as follows:

$$\hat{x}^{(1)}(m) = \left( x^{(0)}(1) - \frac{\hat{\beta}}{\hat{\alpha}} \right) e^{-\hat{\alpha}(m-1)} + \frac{\hat{\beta}}{\hat{\alpha}} \quad ; \quad m = 1, 2, \dots, n, n+1, \dots \quad \dots (8)$$

IV. After all the values of  $\hat{x}^{(1)}$  are obtained, the inverse accumulated generating operation is performed using the following equation [7]:

$$\hat{x}^{(0)}(m+1) = \hat{x}^{(1)}(m+1) - \hat{x}^{(1)}(m) \quad \dots (9)$$

### 3. Gray Wave Forecasting Model

Gray models of one or more variables are useful for predicting the behavior of a monotonous time series, as they give high accuracy predictions, but they are inefficient in the case of time series with cyclical and episodic fluctuations, which prompted researchers to think about predicting the wave of data behavior in the future based on the original data wave. The type of prediction of the wave prediction, which is one of the most important gray predictions and the adopted model is called the gray-wave model [12], the idea of the gray wave forecasting model is summarized in three main steps: selecting the contour lines, determining the time sequences, and applying the gray model GM(1,1) on the time sequences and these steps will be explained as follows [13]:

I. Choosing contour lines: In the gray wave, equal contour lines are originally used based on the maximum and minimum values in the data. This method is suitable when fluctuations are regular, however, it may not be, so Chen et al. [3] suggested unequal contour lines. Now, let's symbolize the original series as follows:

$$X = (x(1), x(2), \dots, x(n)) \quad \dots (10)$$

then we arrange this series in ascending order to get the following series:

$$X^a = (x^a(1), x^a(2), \dots, x^a(n)) \quad \dots (11)$$

Let  $\xi_0 = x^a(1)$ ,  $\xi_s = x^a(n)$ , and  $\xi_1, \xi_2, \dots, \xi_{s-1}$  be s-quantile data, where:

$$\xi_i = \begin{cases} x^a([w]) & w \text{ is not integer} \\ \frac{[x^a(w) + x^a([w+1])]}{2} & w \text{ is integer} \end{cases} \quad \dots (12)$$

Where:

$$w = n(i/s) \quad \dots (13)$$

$\xi_i$  represents the unequal-interval contour lines of the original series.

II. Identifying contour time sequences: which are determined based on the intersects between the unequal interval contour lines  $\xi_i$  obtained from equation (12), and the original time series  $X$ , let  $X_{\xi_i} = (P_1, P_2, \dots, P_{m_i})$  be a set of intersection points,  $P_j$  lies on the  $t_j$  broken line and the coordinates of  $P_j$  are  $(q(j), \xi_j)$ , where [15]:

$$q(j) = t_j + \frac{\xi_j - x(t_j)}{x(t_j + 1) - x(t_j)}, \quad j = 1, 2, \dots, m \quad \dots (14)$$

Thus  $Q_i^{(0)} = (q_i(1), q_i(2), \dots, q_i(m_i))$ ,  $i = 0, 1, 2, \dots, s$  represent the time sequences of the contour line  $\xi_i$ .

III. Filtering contour time sequences and forecasting with GM(1,1): For gray wave forecasting, originally used time sequences that contain 4 or more elements [15], but when the time series fluctuates, this is not convenient to predict but it is necessary to filter these sequences when used with GM(1,1) [2]. The idea of filtering depends on the autocorrelation of the time series, as the autocorrelation measures the similarity between the observations as a function of time, based on it we can choose the time sequences whose ends are close to the sequence of the first future value, then we can use the GM(1,1) model based on the sequences that are selected and as follows [17]:

Let  $t_1^f$  represent the serial number of the first forecasted value and  $Q_i^{(0)} = (q_i(1), q_i(2), \dots, q_i(m_i))$  represent the time sequences chosen for the contour line  $\xi_i$ , for which  $(t_1^f - q_i(m_i))$  is very small. Establish the gray model GM(1,1) based on the chosen time sequences  $Q_i^{(0)}$  to get the forecasting values of

$q_i(m_i + 1), q_i(m_i + 2), \dots, q_i(m_i + k_i)$ . Arranged all the elements in contour time sequences  $Q_0^{(0)}, Q_1^{(0)}, \dots, Q_s^{(0)}$  in ascending order and deleting the invalid values, let  $(q_i(1) < q_i(2) < \dots < q_i(n_s))$  are the forecasting series, such that  $n_s \leq \sum_{i=1}^s (m_i + k_i)$ . If  $q(k)$  is a point located on the contour line  $\xi_{q(k)}$ , the in sample estimated and out sample forecasting of the generated wave is as follows:

$$X = X^{(0)} = \left\{ \xi_{q(k)} + \frac{t - q(k)}{q(k+1) - q(k)} [\xi_{q(k+1)} - \xi_{q(k)}] \right\}, \quad k = 1, 2, \dots, n_s \quad \dots (15)$$

#### 4. Application

The data under research, represented the monthly average of crude oil production, in millions of barrels in Iraq, for the period from January 2016 to December 2021, that is we have a monthly time series of 72 observations. These data are presented in Table (1), and Figure (1) illustrates the plot of this series.

Table (1): Monthly crude oil production from Jan. 2016 to Dec. 2021.

	2016	2017	2018	2019	2020	2021
January	88.3	102.4	129.8	132	135.2	142
February	95.5	85.3	118.1	118	122.1	127
March	95.7	111.6	125.9	132	135.1	140
April	91.9	103.2	121.9	126	130.8	135
May	98.5	110.1	125.5	131	135.2	142
June	93.3	115.7	123.2	126	130.8	138
July	98.9	123.5	128.3	132	138.3	143
August	101.8	124.8	129.9	132	138.3	144
September	103.8	120.6	126.6	129	133.8	139
October	102.8	121.6	131.6	131	138.3	142
November	98.1	120.3	128.5	124	133.6	138
December	112.2	128.1	135.9	130	138.6	141

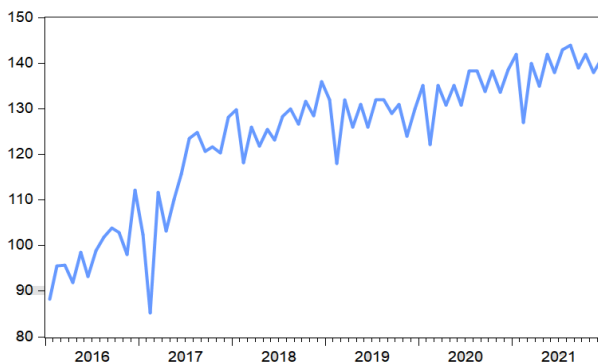


Figure (1): Monthly crude oil production from Jan. 2016 to Dec. 2021.

We note from Figure 1 that the series is fluctuant, and this is supported by the descriptive statistics related to it, as the difference between the maximum value and the minimum value is relatively large, and the results of the Jarque-Bera test indicate that these data do not follow the Normal distribution and the following table shows these measures:

Table (2): Description statistics of monthly crude oil production.

Mean	Max	Min	Std. Dev.	Jarque-Bera Test
123.5736	144	85.3	15.23413	9.494792 (0.008674)

Table 3 shows the autocorrelation coefficients from the first lag to the tenth lag, which indicate that this series is highly auto-correlative, so we can choose the time sequences from it.

Table (3): Autocorrelation analysis of crude oil production.

Lag	1	2	3	4	5
ACF	0.834	0.820	0.783	0.710	0.675
prob.	0.000	0.000	0.000	0.000	0.000
Lag	6	7	8	9	10
ACF	0.587	0.583	0.523	0.495	0.465
prob.	0.000	0.000	0.000	0.000	0.000

For prediction purposes, this series consisting of 72 observations has been divided into two parts, the first part for the period from January 2016 to December 2020, meaning that it is 60 observations, and the second part is 12 observations for the period between January and December of 2021. The first part was used to select the contour lines and determine the time sequences, and the second part was used to evaluate the prediction, and the prediction in this research is a prediction of 12 steps.

Now we start with the first step of the gray wave prediction, which is represented by choosing the contour lines, where  $s=9$  was used, meaning there are ten contour lines as shown in figure (2), Table (4) also shows the number of broken lines that intersect each of the contour lines. while table (5) presents the time sequences which represent the second step.

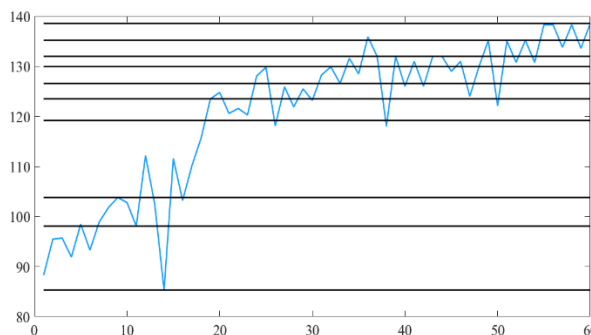


Figure (2): Ten contour lines for crude oil production.

Table (4): Contour lines and their intersection points.

$i$	0	1	2	3	4	5	6	7	8	9
$\xi_i$	85.3	98.1	103.8	119.2	123.5	126.6	129.95	132	135.2	138.6
$m_i$	1	6	6	5	13	14	17	12	10	1
$t_i$	14	4	9	18	19	23	25	35	35	60
		5	11	25	20	25	32	37	36	
		6	12	26	23	30	33	39	49	
		11	14	37	25	33	34	43	51	
		13	15	38	26	37	35	44	53	
		14	16		27	38	37	48	54	
					28	39	38	49	56	
					29	40	39	50	57	
					30	41	40	51	58	
					37	42	41	52	59	
					38	46	42	53		
					49	47	44	54		
					50	49	45			
						50	46			
							48			
							49			
							50			

Table (5): Contour time sequences.

$i$	0	1	2	3	4	5	6	7	8	9
$Q_i^{(0)}$	14.00	4.94	9.00	18.45	19.00	23.81	24.99	35.47	35.91	60.00
		5.08	11.40	25.91	20.31	25.27	31.98	37.00	36.18	
		6.86	12.86	26.14	23.41	30.67	33.67	39.00	49.00	
		11.00	14.70	37.91	25.54	33.00	34.53	43.00	50.98	
		13.25	15.93	38.09	26.69	37.39	35.20	44.00	53.00	
		14.49	16.09		27.60	38.61	37.15	48.38	54.59	
					28.44	39.90	38.85	49.24	56.69	
					29.87	40.12	39.34	50.76	57.31	
					30.06	41.88	40.79	51.72	58.66	
					37.61	42.10	41.21	52.27	59.32	
					38.39	46.63	42.66	53.73		
					49.89	47.43	44.68	54.16		
					50.11	49.66	45.47			
						50.35	46.15			
							47.99			
							49.40			
							50.60			

Now, we start with the third step, which represents the filtering of the time sequences and the formation of the GM(1,1) model, we know that the serial number of the first future value is 61, and the series showed strong autocorrelation as shown in Table (3), and  $Q_6^{(0)}, Q_7^{(0)}$  and  $Q_8^{(0)}$  in GM(1,1) modeling because it is the closest to the future value, and the accumulative generation operator will be found according to equation (2) for all of these sequences, as shown in table (6).

Table (6): Accumulating generator operators.

$i$	6	7	8
<b>AGO</b>	24.99	35.47	35.91
	56.97	72.47	72.08
	90.64	111.47	121.08
	125.17	154.47	172.06
	160.37	198.47	225.06
	197.52	246.86	279.65
	236.37	296.10	336.34
	275.71	346.86	393.65
	316.50	398.58	452.31
	357.71	450.86	511.63
	400.37	504.58	
	445.05	558.74	
	490.53		
	536.68		
	584.67		
	634.07		
	684.67		

Then we get the values of  $z^{(1)}(m)$  according to equation (4) and as in the following table:

Table (7). The mean generation of consecutive neighbors' sequences.

$i$	6	7	8
<b><math>z_i^{(1)}</math></b>	40.98	53.97	54.00
	73.81	91.97	96.58
	107.91	132.97	146.57
	142.77	176.47	198.56
	178.94	222.67	252.35
	216.94	271.48	307.99
	256.04	321.48	364.99
	296.11	372.72	422.98
	337.11	424.72	481.97
	379.04	477.72	
	422.71	531.66	
	467.79		
	513.60		
	560.67		
	609.37		
	659.37		

Using least squares method, the following estimates were obtained:

$$\hat{p}_6 = \begin{bmatrix} \hat{\alpha}_6 \\ \hat{\beta}_6 \end{bmatrix} = \begin{bmatrix} -0.0295 \\ 31.534 \end{bmatrix}$$

$$\hat{p}_7 = \begin{bmatrix} \hat{\alpha}_7 \\ \hat{\beta}_7 \end{bmatrix} = \begin{bmatrix} -0.0354 \\ 37.67 \end{bmatrix}$$

$$\hat{p}_8 = \begin{bmatrix} \hat{\alpha}_8 \\ \hat{\beta}_8 \end{bmatrix} = \begin{bmatrix} -0.0417 \\ 42.09 \end{bmatrix}$$

After getting the values of  $\hat{\alpha}$  and  $\hat{\beta}$  the response time values are calculated as follows:

$$q_6^{(1)}(m) = \left( x^{(0)}(1) - \frac{\hat{\beta}_6}{\hat{\alpha}_6} \right) e^{-\hat{\alpha}_6(m-1)} + \frac{\hat{\beta}_6}{\hat{\alpha}_6} = 1093.939e^{0.0295(m-1)} - 1068.949$$

$$q_7^{(1)}(m) = \left( x^{(0)}(1) - \frac{\hat{\beta}_7}{\hat{\alpha}_7} \right) e^{-\hat{\alpha}_7(m-1)} + \frac{\hat{\beta}_7}{\hat{\alpha}_7} = 1099.594e^{0.0354(m-1)} - 1064.124$$

$$q_8^{(1)}(m) = \left( x^{(0)}(1) - \frac{\hat{\beta}_8}{\hat{\alpha}_8} \right) e^{-\hat{\alpha}_8(m-1)} + \frac{\hat{\beta}_8}{\hat{\alpha}_8} = 1045.263e^{0.0417(m-1)} - 1009.353$$

then:

$$q_i(m) = q_i^{(1)}(m) - q_i^{(1)}(m-1)$$

After the estimated values are obtained, we can predict the time series, and the value of the root mean squares error criterion was calculated and its value was equal to 3.0723, the original series and the predictive series was drawn together in the following figure, which shows the extent of convergence between them.

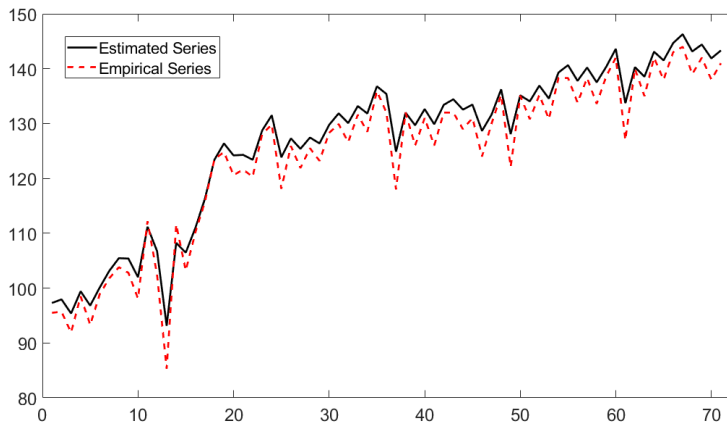


Figure (3): The estimated and the empirical series.

## 5. Conclusion

In this research, the monthly rate of crude oil production in Iraq was studied, and the results showed that this series is not a monotonous series, and had irregular fluctuation ranges, the results also showed that these data do not follow the Normal distribution and that the autocorrelation is strong. The Contour time sequences (6, 7, 8) were chosen and used to predict the series because they are the closest to the first future value. A graph showed how close the original empirical series was to what was estimated by the gray wave method.

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