Statistics Canada Statistique Canada

Environmental Input-Output Modelling

Workshop on the System of Environmental-Economic Accounting Central Framework and Sustainable Development Goals indicators

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Amman, Jordan

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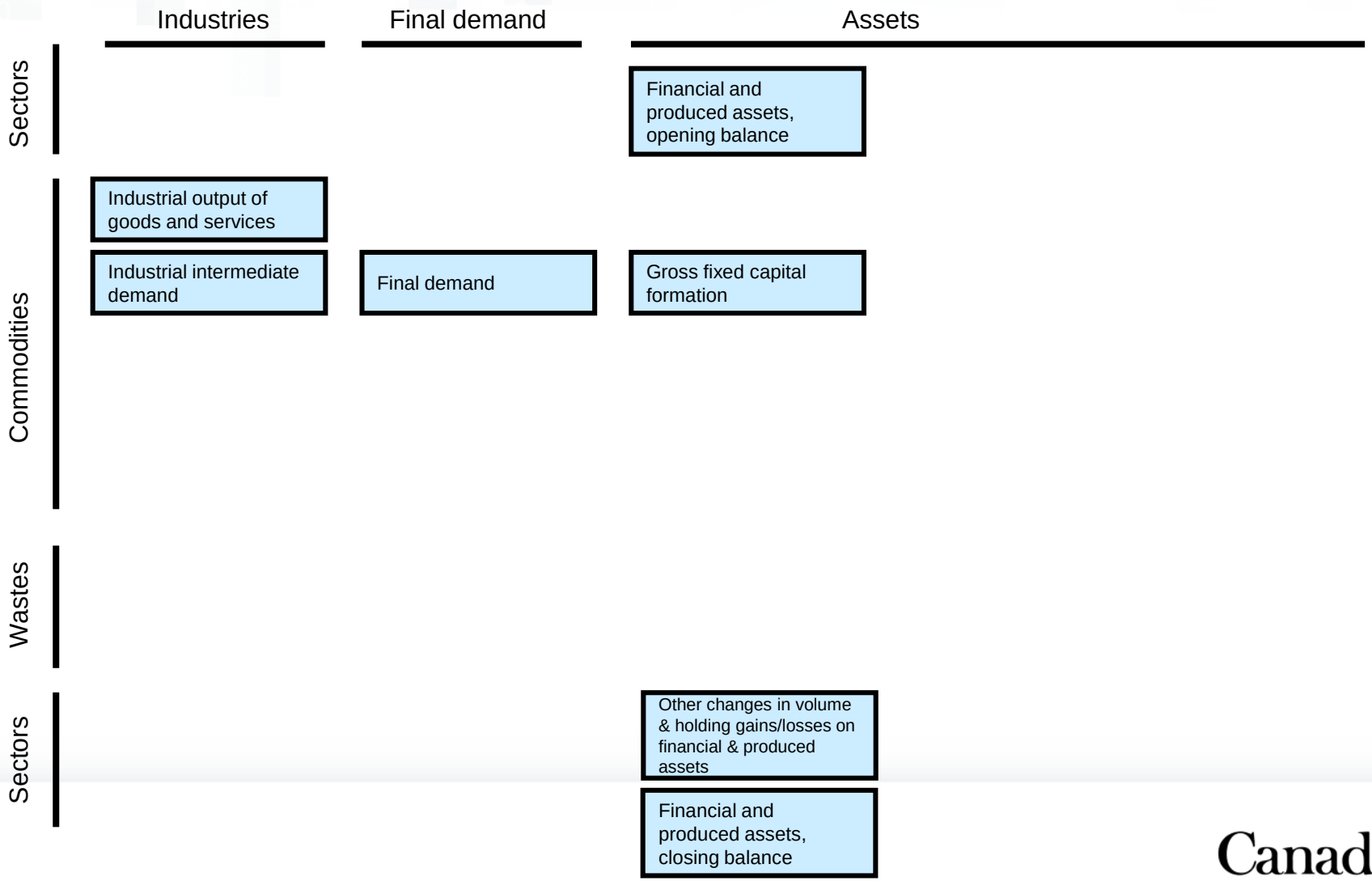
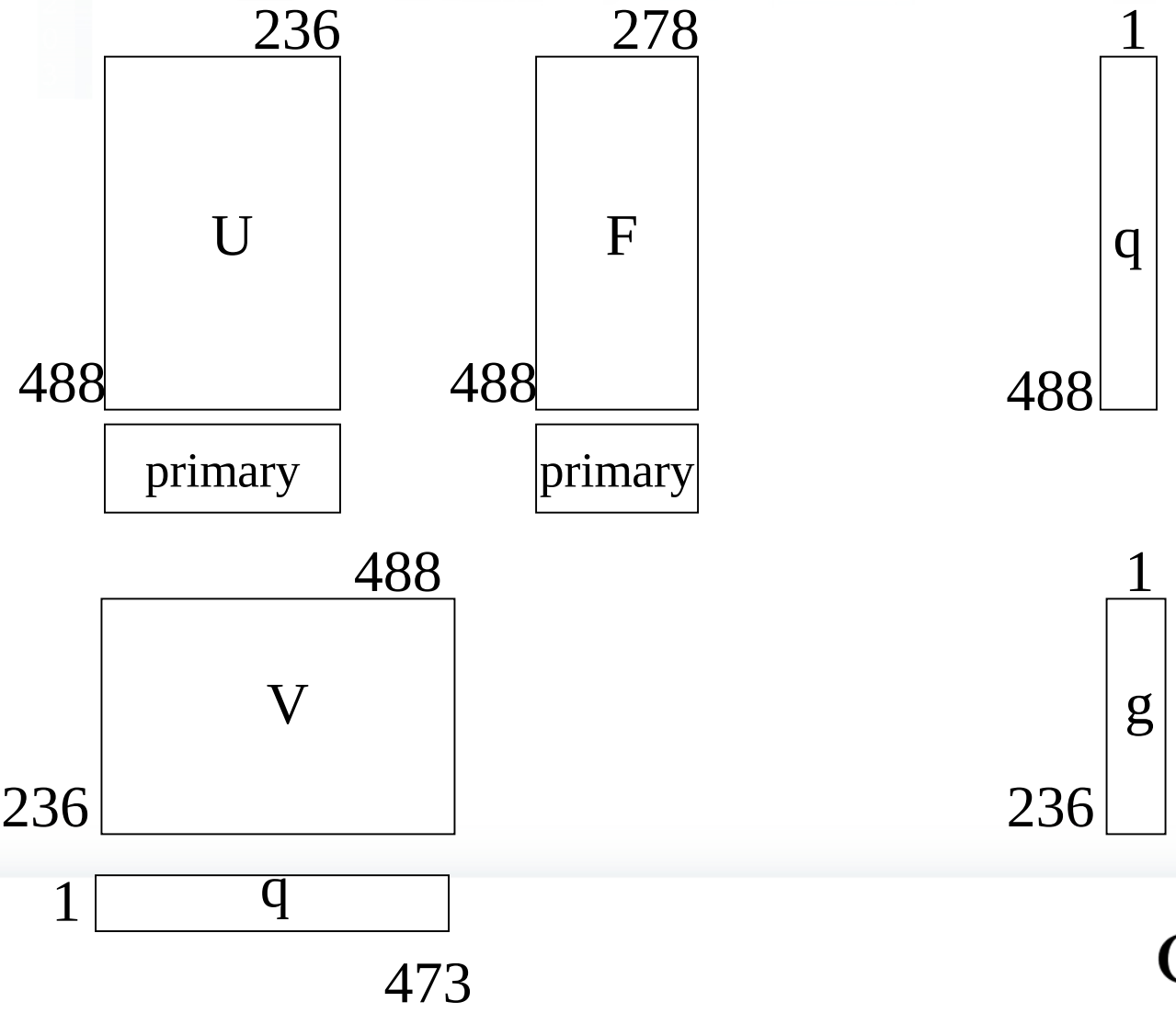
Statistics Canada

Canada

SNA framework



Monetary Supply and Use Tables



Numerical example (U: Use)

USE (U)	farms	mines	food manuf.	other manuf.	services
cattle	10	0	80	10	0
iron ore	0	0	0	100	0
milk	10	5	100	0	5
cheese	0	5	0	0	5
fuel	100	200	10	50	50
steel	0	5	0	145	0
cars	10	5	5	5	10
advertising	5	15	20	40	20
Primary inputs	75	865	65	10	10

Numerical example (V: Supply)

USE (U)	farms	mines	food manuf.	other manuf.	services
cattle	10	0	80	10	0
iron ore	0	0	0	100	0
milk	10	5	100	0	5
cheese	0	5	0	0	5
fuel	100	200	10	50	50
steel	0	5	0	145	0
cars	10	5	5	5	10
advertising	5	15	20	40	20
Primary inputs	75	865	65	10	10

SUPPLY (V)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	100	0	100	10	0	0	0	0
mines	0	100	0	0	1000	0	0	0
food manuf.	0	0	80	200	0	0	0	0
other manuf.	0	0	0	0	10	150	200	0
services	0	0	0	0	0	0	0	100

Numerical example (F: final demand)

USE (U)	farms	mines	food manuf.	other manuf.	services
cattle	10	0	80	10	0
iron ore	0	0	0	100	0
milk	10	5	100	0	5
cheese	0	5	0	0	5
fuel	100	200	10	50	50
steel	0	5	0	145	0
cars	10	5	5	5	10
advertising	5	15	20	40	20
Primary inputs	75	865	65	10	10

Final Demand
0
0
60
200
600
0
165
0

SUPPLY (V)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	100	0	100	10	0	0	0	0
mines	0	100	0	0	1000	0	0	0
food manuf.	0	0	80	200	0	0	0	0
other manuf.	0	0	0	0	10	150	200	0
services	0	0	0	0	0	0	0	100

Numerical example

(g: gross industry output / q: gross commodity output)

USE (U)	farms	mines	food manuf.	other manuf.	services
cattle	10	0	80	10	0
iron ore	0	0	0	100	0
milk	10	5	100	0	5
cheese	0	5	0	0	5
fuel	100	200	10	50	50
steel	0	5	0	145	0
cars	10	5	5	5	10
advertising	5	15	20	40	20
Primary inputs	75	865	65	10	10

Final Demand
0
0
60
200
600
0
165
0

SUPPLY (V)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	100	0	100	10	0	0	0	0
mines	0	100	0	0	1000	0	0	0
food manuf.	0	0	80	200	0	0	0	0
other manuf.	0	0	0	0	10	150	200	0
services	0	0	0	0	0	0	0	100

Σ

g
210
1100
280
360
100

Σ

q	100	100	180	210	1010	150	200	100

Numerical example (overview)

USE (U)	farms	mines	food manuf.	other manuf.	services
cattle	10	0	80	10	0
iron ore	0	0	0	100	0
milk	10	5	100	0	5
cheese	0	5	0	0	5
fuel	100	200	10	50	50
steel	0	5	0	145	0
cars	10	5	5	5	10
advertising	5	15	20	40	20
Primary inputs	75	865	65	10	10

Final Demand
0
0
60
200
600
0
165
0

SUPPLY (V)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising	g
farms	100	0	100	10	0	0	0	0	210
mines	0	100	0	0	1000	0	0	0	1100
food manuf.	0	0	80	200	0	0	0	0	280
other manuf.	0	0	0	0	10	150	200	0	360
services	0	0	0	0	0	0	0	100	100
q	100	100	180	210	1010	150	200	100	

Σ

Σ

The Input-Output model

- ***Basic identity: supply = demand***

$$q + inv_{-} + m = u + fd + x + inv_{+}$$

By substituting for market share ($D=V/q$) and technology ($B=U/g$), we get:

$$g = (I - DB)^{-1}De$$

- ***Allows an estimate of the gross production (g) required from each industry to satisfy a given final demand (e) based on pre-defined relationships of market-share (D) and technology (B)***

Numerical example (B: Technology)

B (U/g)	farms	mines	food manuf.	other manuf.	services
cattle	0.05	0.00	0.29	0.03	0.00
iron ore	0.00	0.00	0.00	0.28	0.00
milk	0.05	0.00	0.36	0.00	0.05
cheese	0.00	0.00	0.00	0.00	0.05
fuel	0.48	0.18	0.04	0.14	0.50
steel	0.00	0.00	0.00	0.40	0.00
parts	0.05	0.00	0.02	0.01	0.10
advertising	0.02	0.01	0.07	0.11	0.20
Σ	0.6	0.2	0.8	0.97	0.9

Numerical example (D: Market share)

B (U/g)	farms	mines	food manuf.	other manuf.	services
cattle	0.05	0.00	0.29	0.03	0.00
iron ore	0.00	0.00	0.00	0.28	0.00
milk	0.05	0.00	0.36	0.00	0.05
cheese	0.00	0.00	0.00	0.00	0.05
fuel	0.48	0.18	0.04	0.14	0.50
steel	0.00	0.00	0.00	0.40	0.00
parts	0.05	0.00	0.02	0.01	0.10
advertising	0.02	0.01	0.07	0.11	0.20
Σ	0.6	0.2	0.8	0.97	0.9

D (V/q)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	1.00	0.00	0.56	0.05	0.00	0.00	0.00	0.00
mines	0.00	1.00	0.00	0.00	0.99	0.00	0.00	0.00
food manuf.	0.00	0.00	0.44	0.95	0.00	0.00	0.00	0.00
other manuf.	0.00	0.00	0.00	0.00	0.01	1.00	1.00	0.00
services	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
Σ	1	1	1	1	1	1	1	1

Numerical example (The inverse)

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

Meaning: dollars of output from industry at row to deliver (to final demand) a dollar of output from industry at column

Numerical example (De)

D (V/q)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	1.00	0.00	0.56	0.05	0.00	0.00	0.00	0.00
mines	0.00	1.00	0.00	0.00	0.99	0.00	0.00	0.00
food manuf.	0.00	0.00	0.44	0.95	0.00	0.00	0.00	0.00
other manuf.	0.00	0.00	0.00	0.00	0.01	1.00	1.00	0.00
services	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

	Final Demand
cattle	0
iron ore	0
milk	60
cheese	200
fuel	600
steel	0
parts	165
advertising	0

Matrix Multiplication

$$A \cdot B = C$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \cdot \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \\ b_{31} & b_{32} \end{bmatrix} = \begin{bmatrix} c_{11} = a_{11}* b_{11} + a_{12}* b_{21} + a_{13}* b_{31} & c_{12} = a_{11}* b_{12} + a_{12}* b_{22} + a_{13}* b_{32} \\ c_{21} = a_{21}* b_{11} + a_{22}* b_{21} + a_{23}* b_{31} & c_{22} = a_{21}* b_{12} + a_{22}* b_{22} + a_{23}* b_{32} \end{bmatrix}$$

$$(2 \times 3) \cdot (3 \times 2) = (2 \times 2)$$

Numerical example (De)

D (V/q)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	1.00	0.00	0.56	0.05	0.00	0.00	0.00	0.00
mines	0.00	1.00	0.00	0.00	0.99	0.00	0.00	0.00
food manuf.	0.00	0.00	0.44	0.95	0.00	0.00	0.00	0.00
other manuf.	0.00	0.00	0.00	0.00	0.01	1.00	1.00	0.00
services	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

	Final Demand
cattle	0
iron ore	0
milk	60
cheese	200
fuel	600
steel	0
parts	165
advertising	0

(industry x commodity) • (commodity x demand) = (industry x demand)

Numerical example (De)

D (V/q)	cattle	iron ore	milk	cheese	fuel	steel	parts	advertising
farms	1.00	0.00	0.56	0.05	0.00	0.00	0.00	0.00
mines	0.00	1.00	0.00	0.00	0.99	0.00	0.00	0.00
food manuf.	0.00	0.00	0.44	0.95	0.00	0.00	0.00	0.00
other manuf.	0.00	0.00	0.00	0.00	0.01	1.00	1.00	0.00
services	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00

	Final Demand
cattle	0
iron ore	0
milk	60
cheese	200
fuel	600
steel	0
parts	165
advertising	0

De	
farms	42.86
mines	594.06
food manuf.	217.14
other manuf.	170.94
services	0.00

= 60* 0.56 + 200* 0.05

Actually = 60*0.555 + 200*0.0476

(display table is rounded)

Numerical example: $(I-DB)^{-1}De$

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

De	
farms	42.86
mines	594.06
food manuf.	217.14
other manuf.	170.94
services	0.00

Numerical example: $(I-DB)^{-1}De$

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

De	
farms	42.86
mines	594.06
food manuf.	217.14
other manuf.	170.94
services	0.00

47.5114
7.0069
141.185
14.297
0
210

g=$(I-DB)^{-1}De$
210
1100
280
360
100

decompostion of row one of $(I-DB)^{-1}$ times column one of De (Row 1 of $(I-DB)^{-1}$ times column of De)

1.11 * 42.86 ($1.11 of production from farms is required to deliver a dollar of production from farms, so to get 42.86 of final demand we need farms to produce 47.5 dollars of output)

Plus: 0.01 * 594.06 ($0.01 of production from farms is required to deliver a dollar of production from mines, so to get 594.06 of final demand we need farms to produce 7 dollars of output)

Plus: 0.65 * 217.14 ($0.65 of production from farms is required to deliver a dollar of production from food manufacturers, so to get 217.14 of final demand we need farms to produce 141.2 dollars of output)

Plus: 0.08 * 170.94 ($0.08 of production from farms is required to deliver a dollar of production from other manufacturers, so to get 170,94 of final demand we need farms to produce 14.3 dollars of output)

Plus: 0.12 * 0 ($0.12 of production from farms is required to deliver a dollar of production from services, so to get 0.00 to final demand we need farms to produce 0 dollars of output)

So, to deliver all of final demand, farms must produce 210 dollars of gross output

Numerical example

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

De	
farms	42.86
mines	594.06
food manuf.	217.14
other manuf.	170.94
services	0.00

g calc	$g=(I-DB)^{-1}De$
farms	210
mines	1100
food manuf.	280
other manuf.	360
services	100

real g	g
farms	210
mines	1100
food manuf.	280
other manuf.	360
services	100

Calculating output required for different categories of demand...

$$g = (I - DB)^{-1}De$$

- *Recall that this provides an estimate of the gross output (g) required from each industry to satisfy a given final demand (e) based on pre-defined relationships of market-share (D) and technology (B)*

- *So, if I wanted to calculate the gross output from each industry required to produce the demand related to household personal expenditure, I could calculate...*

$$g_{pe} = (I - DB)^{-1}De_{pe}$$

Integration – getting the environment in there...

$$i = E/g$$

$$\overset{1}{\underset{235}{\boxed{i}}} = \overset{1}{\underset{235}{\boxed{E}}} \Big/ \overset{1}{\underset{235}{\boxed{g}}}$$

Sample results – attribution to demand

$$g_{pe} = (I - DB)^{-1} De_{pe}$$

$$\begin{matrix} 1 \\ 1 \end{matrix} \boxed{i}_{235} * \begin{matrix} 1 \\ 235 \end{matrix} \boxed{g_{pe}} = \begin{matrix} 1 \\ 1 \end{matrix} \boxed{E_{pe}}$$

Sample results – attribution to demand

Table 153-0129 [1, 2, 5, 10, 11]

Physical flows by final demand category

annual

Data table | Add/Remove data | Manipulate | Download | Related information | Help

The data below is a part of CANSIM table 153-0129. Use the Add/Remove data tab to customize your table.

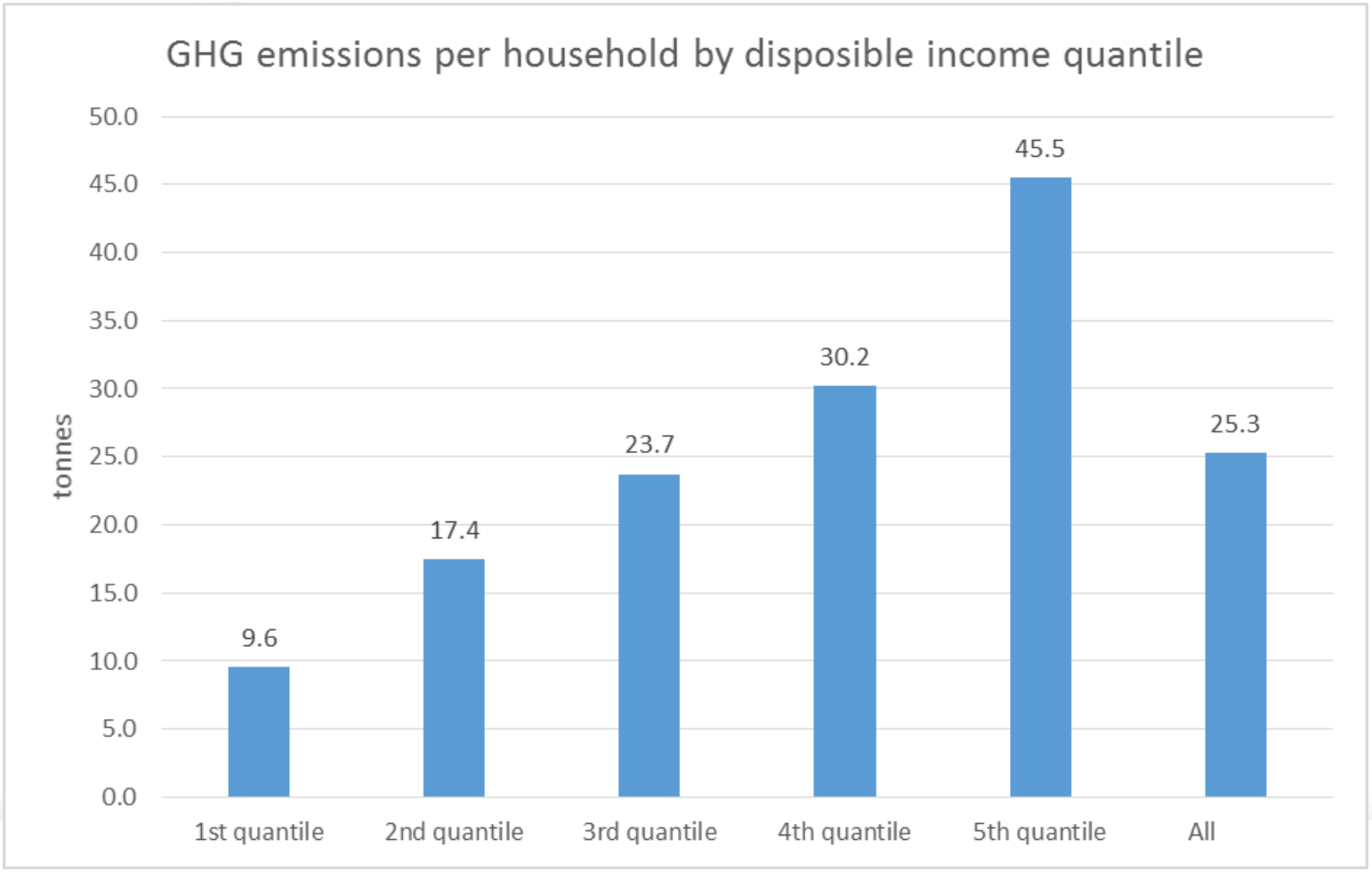
Selected items [Add/Remove data]

Geography = Canada

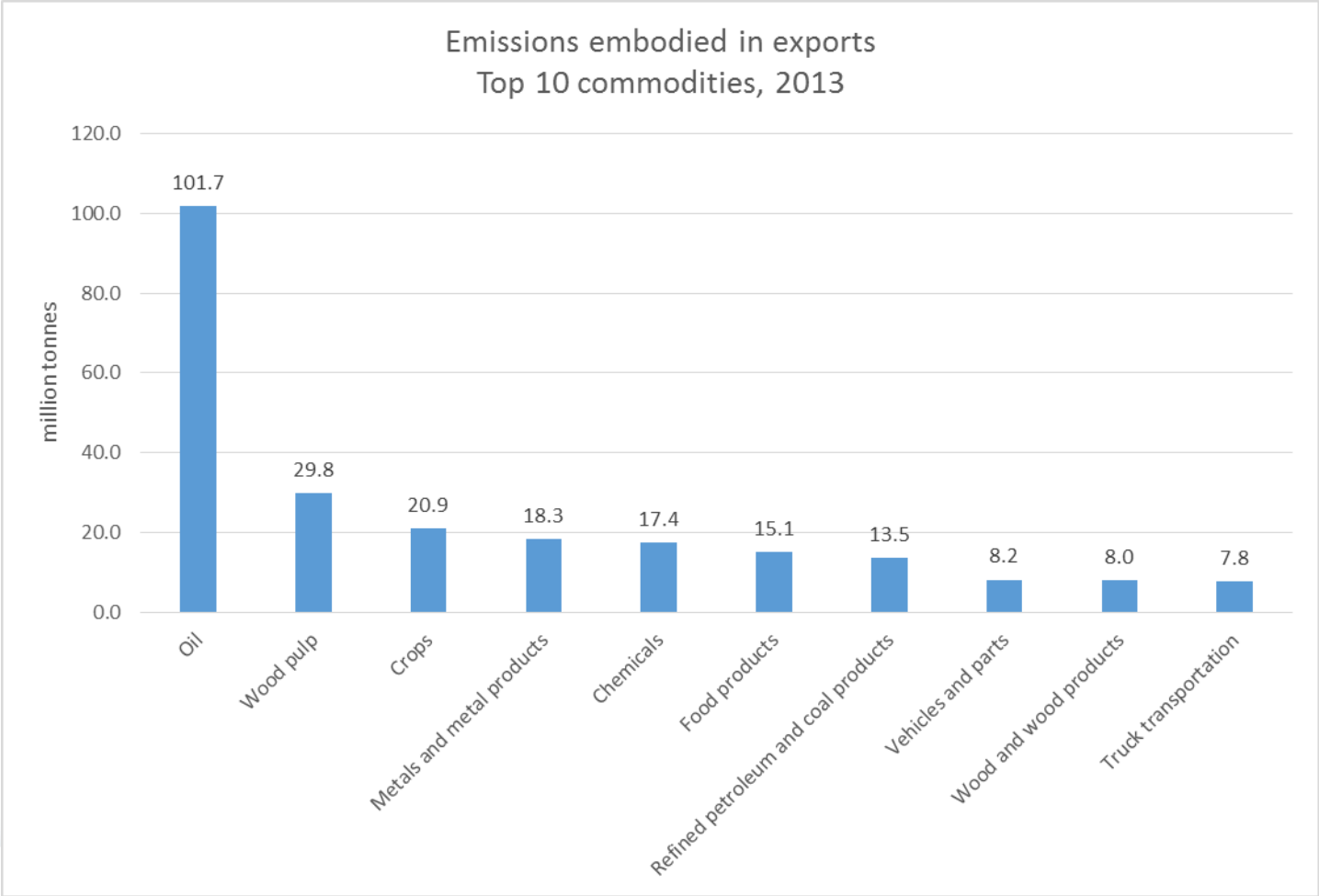
Flow = Greenhouse gas emissions by final demand category (kilotonnes) [5]

Sector	2009	2010	2011	2012	2013
Total, industries and households	710,959	727,176	734,844	742,364	758,467
Personal expenditure (households) [4]	325,729	328,190	326,039	321,016	326,677
Non-profit institutions serving households' consumption expenditure	6,071	5,538	5,626	5,437	5,344
Government net current expenditure	42,692	41,822	41,573	40,495	37,696
Gross fixed capital formation	73,355	80,769	81,610	84,722	84,324
International exports	263,112	270,856	279,996	290,694	304,426

Sample results – attribution to demand: household detail



Sample results – attribution to demand: commodity detail



Sample results – direct and indirect intensities (multipliers)

$$\begin{matrix}1 \\ \end{matrix}\begin{bmatrix} i \end{bmatrix}_{236} * \begin{bmatrix} (I - DB)^{-1} \end{bmatrix}^{236}_{236} =$$

Table 153-0115 [1, 2, 3, 4, 5, 6, 8, 9, 11, 12]

Direct plus indirect energy and greenhouse gas emissions intensity, by industry

annual

Data table | Add/Remove data | Manipulate | Download | Related information | Help

The data below is a part of CANSIM table 153-0115. Use the Add/Remove data tab to customize your table.

Selected items [Add/Remove data]

Geography = Canada
Intensity = Direct plus indirect energy intensity (gigajoules per thousand current dollars of production)

Sector	2013
Total, industries	5.60
Crop and animal production [BS11A00]	10.39
Forestry and logging [BS11300]	8.86
Fishing, hunting and trapping [BS11400]	7.08
Support activities for agriculture and forestry [BS11500]	7.05
Oil and gas extraction [BS21100]	14.80
Coal mining [BS21210]	8.79
Metal ore mining [BS21220]	7.84
Non-metallic mineral mining and quarrying [BS21230]	6.89
Support activities for mining and oil and gas extraction [BS21300]	5.92
Electric power generation, transmission and distribution [BS22110]	24.13
Natural gas distribution, water, sewage and other systems [BS221A0]	4.38
Residential building construction [BS23A00]	4.45
Non-residential building construction [BS23B00]	3.67
Transportation engineering construction [BS23C10]	5.22
Oil and gas engineering construction [BS23C20]	4.06
Electric power engineering construction [BS23C30]	3.31
Communication engineering construction [BS23C40]	5.15
Other engineering construction [BS23C50]	4.45
Repair construction [BS23D00]	3.50
Other activities of the construction industry [BS23E00]	5.15
Animal food manufacturing [BS31110]	8.04

Numerical example: Multipliers

emissions per unit of output (direct emissions intensity)					
α	farms	mines	food manu	other manu	services
co2/g	0.5	0.8	0.3	0.4	0.1

$(I-DB)^{-1}$	farms	mines	food manuf.	other manuf.	services
farms	1.11	0.01	0.65	0.08	0.12
mines	0.74	1.26	0.59	1.13	1.01
food manuf.	0.04	0.01	1.22	0.03	0.12
other manuf.	0.13	0.03	0.14	1.80	0.27
services	0.07	0.03	0.16	0.27	1.32

alpha inverse	farms	mines	food manuf.	other manuf.	services
co2/g	1.22	1.03	1.23	1.70	1.14

Numerical example: Multipliers

emissions per unit of output (direct emissions intensity)											
α	farms	mines	food manuf.	other manuf	services		(I-DB)$^{-1}$	farms	mines	food manu	other manu	services
co2/g	0.5	0.8	0.3	0.4	0.1		farms	1.11	0.01	0.65	0.08	0.12
							mines	0.74	1.26	0.59	1.13	1.01
							food manu	0.04	0.01	1.22	0.03	0.12
alpha inverse	farms	mines	food manuf.	other manuf	services		other manu	0.13	0.03	0.14	1.80	0.27
co2/g	1.22	1.03	1.23	1.70	1.14		services	0.07	0.03	0.16	0.27	1.32

So, α*(I-DB)$^{-1}$ for row of α times column 1 of (I-DB)$^{-1}$ is...		
	0.5 * 1.11 (1.11 dollars of production from farms is required to deliver a dollar of production from farms.	
	Farms emit .05 CO2 per dollar of output,so in terms of emissions this is:)	0.5543
plus	0.8 * 0.74 (0.74 dollars of production from mines is required to deliver a dollar of production from farms.	
	Mines emit .8 CO2 per dollar of output, so in terms of emissions this is:)	0.59495
plus	0.3 * 0.04 (0.04 dollars of production from food manuf. Is required to deliver a dollar of production from farms.	
	Food manuf. Emit .3 CO2 per dollar of output, so in terms of emissions:)	0.01171
plus	0.4 * 0.13 (0.13 dollars of production from other manuf. Is required to deliver a dollar of output from farms.	
	Other manuf. Emit .4 CO2 per dollar of output, so in terms of emissions:)	0.05075
plus	0.1 * 0.01 (0.01 dollars of production from services is required to deliver a dollar of output from farms.	
	Services emit .1 CO2 per dollar of output, so in terms of emissions this is:)	0.00668
	This is the total emissions required (direct plus indirect) from all industries per dollar of output from farms.	**1.21839**

this basically converts emissions per unit of output (direct intensity) to total emissions required to deliver a unit of final demand (direct plus indirect intensity)

Uses of the Input-Output model

- *Demand based analysis*
- *Decomposition analysis*
  - *Attribution environmental change to changes in:*
    - *Demand patterns*
    - *Industry production functions*
- *Analysis of impacts of:*
  - *Trade agreements*
  - *Policy implementation (e.g. carbon taxes)*

Questions?

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